CORRECTION FOR STOKES' LAW

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ABSTRACT. Millikan's correction to Stokes' law for water droplets falling through air has been measured. Some necessary improvements over Millikan's method have been introduced. The present direct measurement yields the value $A = 0.701$ for the relevant correction constant

INTRODUCTION

To study the rate of evaporation of unattached water droplets, they have been allowed to evaporate at a controlled temperature and pressure within Millikan's oil-drop apparatus, containing air at some specified relative humidity With the help of Stokes' law, the droplet radii have been calculated by measuring the values of velocity under gravity at intervals in the usual manner. The range of the droplet radii in the present series of experiments varies from 5×10^{-5} cm to 2×10^{-4} cm. Since such droplet radii are not large compared to the mean free path in air at room temperature and atmospheric pressure, Millikan's correction to Stokes' law has been experimentally determined for water droplets falling through air.

MILLIKAN'S CORRECTION TO STOKES' LAW

In gases at ordinary pressures the slip due to molecular inhomogeneities makes Stokes' law slightly incorrect even for spheres with radii as large as 0.005 mm Millikan has suggested that the usual Stokes' law should be replaced by an equation of the general form,

$$X = 6\pi\eta R V_g(1+A'l/R)^{-1} \quad \ldots \quad (1)$$

where l is the molecular mean free path in air and

$$A' = A+Be^{-CR/l} \quad \ldots \quad (2)$$

where A and B are constants depending on the nature of the gas and the drop substance, while C is equal to 1.25 Millikan determined the values of A' by the oil-drop method, for a wide range of the ratio l/R, for different drop substances and media.

The correction $(1+A'l/R)^{-1}$ can also be written as $(1+b/pR)^{-1}$, where b is a constant and p is the pressure.

DETERMINATION OF A' FOR WATER DROPLETS FLLING THROUGH AIR

It may be noted that as the readings in the present series of experiments are taken at atmospheric pressure, the second term in equation (2) would become quite negligible and A' is equal to A

The higher rate of evaporation of water, as compared to that of oil, makes the application of Millikan's procedure to determine the value of A' for water rather difficult. In order to avoid any appreciable evaporation of the observed droplet, the following three precautions have been taken:

(i) The air inside the chamber was made saturated with water vapour by keeping four small water-boats inside this chamber

(ii) With the above arrangement the air inside the chamber is made saturated for plane water surface. However, small drops with radii of the order of 10^{-4} to 10^{-5} cm will evaporate as the vapour pressure on a convex surface is more than that on a plane surface Hence before

TABLE I

Observed relation between $(e_1^{2/3})$ and $(1/pR)$

Observat numbe	t_g (sec)	t_f (sec)	$v_g \times 10^3$ (cm/sec)	$v_f \times 10^3$ (cm/sec)	$e_n \times 10^{10}$ (e.s u.)	n	$e_1 \times 10^{10}$ (e.s.u.)	$e_1^{2/3} \times 10^6$	$R \times 10^5$ (cm)	p (cm of Hg)	$1/pR$
1	2	3	4	5	6	7	8	9	10	11	12
1	40.0	3.2	2.025	25.31	17 91	3	5.970	70 89	4 142	76.40	316.0
2	25.3	4.5	3 202	18.00	17.46	3	5.820	69.69	5.208	76.30	251.7
3	8.5	14.7	9.530	5.511	21.36	4	5.340	66.20	8.984	76.40	145.6
4	14 0	8.8	5.786	9 204	16.60	3	5.533	67.39	7.001	76.35	187 1
5	30.7	6.2	2 638	13.06	11 73	2	5.865	70 06	4 729	76.45	276.7
6s	21.5	5 3	3.768	15.29	17.02	3	5 673	68 52	5.649	76 45	231.5
7	16.0	7.3	5.063	11.10	16.74	3	5.580	67.76	6 549	76.40	199.8
8	41.0	11.0	1.975	7.364	6.042	1	6.042	71.47	4.091	76.40	320.0
9	7.5	24.6	10.80	3.294	21.31	4	5.327	65.70	9.566	76.50	136.7
10	11.9	12.0	6.808	6.750	16.28	3	5.426	66.53	7.600	76.50	172.1
11	9.9	9.9	8.183	8.183	21.54	4	5 385	66.19	8.326	76 35	157.3
12	27.6	6.8	2.935	11.91	11.70	2	5.850	69.95	4.986	76.30	262.8

starting to take observations on a droplet, considerable spraying caused numerous droplets to enter the chamber and evaporate for some time. This procedure further reduced the rate of evaporation of the droplet under observation

(iii) The time for which the droplet was under observation was minimized so as to make its evaporation negligible for all practical purposes. The procedure of taking observations has therefore been altered from that followed by Millikan The velocity under gravity for the droplet was measured and almost immediately its velocity in the opposite direction, with the electric field on, was noted

Only two such observations were recorded for each droplet. The variation in the values of the electronic charge ascertained by preliminary observations, was from 5×10^{-10} e.s.u. to 6×10^{-10} e.s u. for the dorplet radii in the present series of experiments. Hence it was possible to guess correctly the value of n when the same was small. It is for this reason that the value is chosen equal to or less than 4.

A number of preliminary observations were taken to calculate approximately the value of e_1. But only those observations, with n between 1 and 4 were used

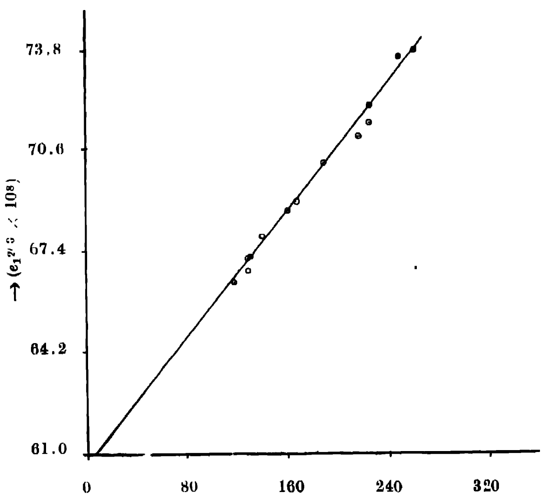

Fig. 1. Variation of $e^{2/3}$ against $(1/pR)$.

for further calculations. These observations are presented in Table I. In making these calculations, the density of the liquid $\sigma = 0.996$ gm cm^{-3}, the density of the medium $\rho = 0.00129$ gm cm^{-3} and the viscosity of the medium $\eta = 183.2 \times 10^{-6}$ dynes per sq. cm. per unit vel. grad. have been used. Further, one division of the tele-microscope equals 0.081 cm. The second column of Table I gives the values of the time t_g taken by the droplet to traverse one division in the tele-microscope under the gravitational field alone. Whereas, the third column gives the values of the time t_f taken by the dorplet to travel one such division when the electric field is applied. After calculating V_g and V_f, e_n is calculated with $F = 2.183$ volts cm^{-1}. The values e_1 and $e_1^{2/3}$ are presented in the eighth and ninth columns respectively The tenth column gives the values of R and the last column gives the values of $(1/pR)$ varying between 136.7 and 320.

The graph showing the variation of $e_1^{2/3}$ against $(1/pR)$ is shown in figure 1. The points lie fairly well on a straight line that passes through the calculated value of $e^{2/3}$ when $(1/pR)$ is zero The constant b was calculated from the slope of this straight line giving $b = 0.000512$ Taking $l = 9.6 \times 10^{-6}$ cm and noting that $A = b/pl$, the value of A was calculated giving $A = 0.701$.

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